# Hygrothermally Induced Nonlinear Free Vibration Response of Nonlinear Elastically Supported Laminated Composite Plates with Random System Properties:

## Stochastic Finite Element Micromechanical Model

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***Abstract***

This paper presents the hygrothermally induced nonlinear free vibration response of nonlinear elastically supported laminated composite plates with random system properties based on micromechanical stochastic finite element model and subjected to uniform temperature and moisture changes. System properties such as material properties of each constituent's material, coefficients of hygroscopic expansion, thermal expansion coefficients and foundation stiffness parameters are taken as independent random input variables for accurate prediction of system behaviour. The basic formulation is based on higher order shear deformation theory (HSDT) with von-Karman nonlinear strains using modified $C^0$ continuity. A direct iterative based nonlinear finite element method in conjunction with first order perturbation technique (FOPT) developed earlier is outlined and applied is extended to solve the stochastic nonlinear generalised eigenvalue problem and to compute the second order statistics (mean and coefficient of variation) of the nonlinear fundamental frequency. Nonlinear free vibration of composite plates subjected to uniform temperature and moisture distribution over plate surface and through the plate thickness is obtained for various combinations of foundation parameters, uniform lateral pressures, staking sequences, fiber volume fractions, aspect ratios, plate thickness ratios, boundary conditions under environmental conditions. The result has been validated with those results available in the literature and independent Monte Carlo simulation (MCS). Some new results are presented also which obviously demonstrate the importance of the randomness in the system parameters on the response of structure in hygrothermal environments.

***Keywords***

*Nonlinear Free Vibration; Micromechanical Model; Stochastic Finite Element; Hygrothermal Environment; Perturbation Technique; Nonlinear Elastic Foundations*

## Introduction

The composite plate/panel subjected to aerodynamic heating such as the external skin of the high speed aircrafts, rockets and launch vehicles etc. is extremely important in the field of aerospace engineering. Aerodynamic heating during supersonic flight induces temperature that changes the configuration of the flights or influences the behaviors of aerospace structures and lowers of the aircraft performance. Actually, moisture and temperature increase and restrain hygrothermal expansion to induce hygrothermal vibration. Hygrothermal effect is important since moisture and temperature change influences the structural behavior such as deformation, buckling loads and natural frequencies. Therefore, there is a need to understand the hygrothermal nonlinear vibration behavior of the composite plates resting on nonlinear elastic foundations under hygrothermal effects. One of the aims of the designer is to control undesirable vibrations that may eventually lead to the failure of the structures due to fatigue and creep. Accurate evaluation of the fundamental frequency is very important. Composite materials have inherent dispersions in material properties due to lack of complete control over the manufacturing and fabrication processes. These materials posses uncertainties in the material properties as compared to their conventional counterparts due to large number of parameters involved in their fabrication. Some important reasons for the property variations of composite material are air entrapment, delamination, lack of resin, interfacial bond characteristics, incomplete curing of resin, excess resin between layers and number

of geometrical parameters involved such as alignment of fibers, volume fractions, inclusions, voids and others. The uncertainties in the system parameters are reflected in uncertainty in its response behavior. It is important for the designer to have an accurate knowledge of the structural response for sensitive applications otherwise the predicted response may differ significantly rendering the structure unsafely. Enhancing accuracy in response evaluation for reliability of design is possible by accounting for the dispersion in system properties in the modelling of the problem, which can be achieved by taking the system properties (such as Young's Modulus, shear modulus, Poisson's ratio and moisture & thermal expansion coefficients) as random basic inputs.

Considerable literature is available on the vibration response of conventional structures and composite structures with randomness in system properties. Leissa and Martin [1990] have analyzed the vibration and buckling of rectangular composite plates and have studied the effects of variation in fiber spacing using classical laminate theory (CLT). Zhang and Chen [1990] have presented a method to estimate the standard deviation of eigen value and eigen vector of random multiple degree of freedom system. Zhang and Ellingwood [1993] have evaluated the effect of random material field characteristics on the instability of a simply supported beam on elastic foundation and a frame using perturbation technique.

A comprehensive summary of extensive literature is available on the response analysis of the structures with deterministic material properties to random excitations Nigam and Narayanan [1992] . However, the analysis of the structures with random system properties is not adequately reported in the literature. Manohar and Ibrahim [1991] have presented excellent reviews of structural dynamic problems with parameter uncertainties. Shankara and Iyenger [1996] studied A $C^0$ element for the free vibration analysis of laminated composite plates. Using CLT in conjunction with FOPT Salim et al. [1998] has studied the static deflection, natural frequency and buckling load of composite rectangular plates with random material properties. Venini and Mariani [2002] have investigated the eigen problem associated with the free vibrations of uncertain composite plates. The elastic moduli of the system, the stiffness of the Winkler foundation on which the plate rests and the mass density are considered to be uncertain. Given their random field-based description, a new method is presented for the computation of the second order statistics of the eigen properties of the laminate. Yadav and Verma [2001] have investigated the free vibration of composite circular cylindrical shells with random material properties employing CLT and FOPT to obtain the second order statistics natural frequencies. Singh et al. [2001, 2003] has analyzed the composite cross-ply laminated composite plate/panel with random material properties for free vibration utilizing higher order deformation theory (HSDT) with FOPT. The mean and variance of the natural frequencies have been obtained for the first five natural frequencies by using exact mean analysis approach and finite element method with a $C^0$ element. Onkar and Yadav [2004] have investigated non-linear response statistics of composite laminates with random material properties under random loading and non-linear free vibration of laminated composite plate with random material properties. Kitiponarchai et al. [2006] has studied the random free vibration response of functionally graded plates with general boundary conditions and subjected to a temperature change to obtain the second order statistics of vibration frequencies. Tripathi et al. [2007] has investigated the free vibration response of laminated composite conical shells with random material properties using FEM in conjunction with FOPT based on HSDT.

Most recently, the present authors investigated the effect of randomness in system properties on elastic buckling and free vibration of laminated composite plates resting on elastic foundation by assuming that the system properties such as Young's modulus, shear modulus, Poisson's ratio and foundation parameters are modeled as random inputs variables Lal et al., [2008]. They used $C^0$ linear and nonlinear FEM based on higher order shear deformation in the presence of small deformation theory and von-Karman large deformation theory in conjunction with mean centered first order perturbation technique to obtain the mean as well as standard deviation of linear and nonlinear frequency response, which was carried out by using macro mechanical model. Whitney and Ashton [1971] investigated the effects of environment on the elastic response of layered composite plates using deterministic finite element method with macro mechanical model. Chen and Chen [1988] found out the vibrations of hygrothermal elastic composite plates using deterministic finite element method to investigate different behavior of plates when exposed to moisture and temperature environments. Ram and Sinha [1992]

researched the hygrothermal effects on the free vibration of laminated composite plates using deterministic finite element approach with first order shear deformation theory. Patel et al. [2002] analyzed the hygrothermal effects on the structural behavior of thick composite laminates using higher-order theory. They found that moisture and temperature significantly affected the thin plates compared to thick plates in buckling, vibration and static bending. Huang and Zheng [2003, 2004] studied the nonlinear vibration and dynamic response to simply supported shear deformable laminated plates on elastic foundations and in hygrothermal environments by using deterministic finite element approach.

It is evident from the available literature that the studies on the hygrothermally induced nonlinear free vibration response to nonlinear elastically supported laminated composite plates with random system properties using micromechanical model are free from full investigation by the researchers. In the present work, an attempt is made to address this problem. The laminated composite plate is analyzed using the $C^0$ nonlinear FEM with HSDT based on von-Karman nonlinear strain-displacement relations. DIFOPT approach has been employed to develop a novel probabilistic solution procedure for random nonlinear problem with a reasonable accuracy. The approach is valid for system properties with random variations in the system variables compared with the mean value. The condition is satisfied by most engineering materials and it is hard to put any limitation on the approach. The efficacy of the probabilistic approach is examined by comparison of the results with an independent Monte Carlo simulation (MCS).

***Formulations***

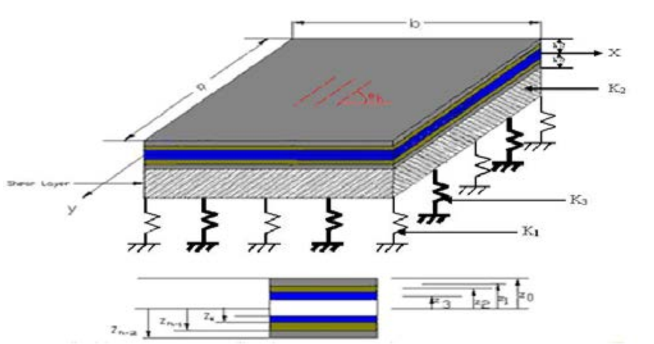

FIG 1. GEOMETRY OF NONLINEAR ELASTICALLY SUPPORTED LAMINATED COMPOSITE PLATE

Consider geometry of laminated composite rectangular plate of length a, width b, and thickness h, which consists of N-plies located in three dimensional Cartesian coordinate system (X, Y, Z) where X- and -Y plane pass through the middle of the plate with thickness of its origin placed at the corner of the plate as shown in Fig. 1.

Let $(\bar{u}, \bar{v}, \bar{w})$ be the displacements parallel to the (X, Y, Z) axes, respectively. The thickness coordinate Z of the top and bottom surfaces of any $k_{th}$ layer are denoted as $Z_{(k-1)}$ and $Z_{(k)}$ respectively. The fiber of the $K_{th}$ layer is oriented with fiber angle $\theta_k$ to the X- axis. The plate is assumed to be subjected to uniformly distributed transverse static load which is defined as $q(x,y) = q_o$.

The plate is assumed to be attached to nonlinear elastic foundation so that no separation takes place in the process of deformation. The load displacement relation between the plate and the supporting foundation is given as [2008]

$$P = K_1 w + K_3 w^3 - K_2 \nabla^2 w \tag{1}$$

where P is the foundation reaction per unit area, and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is Laplace differential operator and $K_1$, $K_2$ and $K_3$ are linear Winkler foundation (normal), linear Pasternak foundation (shear layer) and nonlinear Winkler foundation parameters respectively. This model is simply known as Winkler type when $K_2 = 0$.

The displacement field model, strain displacement, stress – strain relations, strain energy ( Linear and non-linear), kinetic energy, potential energy, external work done and Finite element formulations are reflected in Singh et al.[2001], Shankara and Yadav [1996], Manohar et al. [1991], Chia [1980], Franklin [1968], Reddy [1984], Jones [1998], Kitipornchai et al. [2006] and Lin et al. [2000].

***1) Micromechanical Approach***

The material properties of the fiber composite at different moisture concentration and temperature are evaluated using micromechanical model. Since the effect of temperature and moisture concentration is dominant in matrix material the degradation of the fiber composite material properties is estimated by degradation of the matrix property only. The ratio of the retention of matrix mechanical property is expressed as Upadhyay et al. [2010].

$$F_m = \left[ \frac{T_{gw} - T}{T_{g0} - T_0} \right]^{\frac{1}{2}} \quad (2a)$$

where T=To+ΔT and T- is the temperature at which material property is to be predicted; To is the reference temperature, ΔT= is equal to increase in temperature from reference temperature, Tgw and Tgo are glass transition temperature for wet and reference dry conditions, respectively. The glass transition temperature in wet material is determined as Upadhyay et al. [2010].

The elastic constants are obtained from the following equations Shen [2001].

Similarly, coefficients of hygroscopic expansion are expressed as Shen [2001].

### *The Equation of Motion and its Solution Technique*

#### *1) Governing Equation*

The governing equation for hygrothermally induced nonlinear free vibration of the plate analysis derived from the Hamilton's variational principle expressed as

$$\left[M\right]\{\ddot{q}\}+\left[K\right]\{q\}=0 \quad (3)$$

$$\left[K\right]=\left\{\left[K_l\right]+\left[K_f\right]+\left[K_{nl}\right]-\lambda_{HT}\left[K_g\right]\right\}$$

with

$$\left\{K_{nl}\right\}=\left[\tfrac{1}{2}N_1(q)+\tfrac{1}{3}N_2(q)\right] \quad (4)$$

where $\{q\}$ , $[K_l]$ , $[K_{nl}]$ , $[K_f]$ , $[K_g]$ , $[M]$ and $\lambda_{HT}=\omega^2$ are defined as global displacement vector, global linear stiffness matrix, non-linear stiffness matrix, global foundation matrix geometric stiffness matrix, global mass matrix and critical buckling temperature respectively. $N_1(q)$ and $N_2(q)$ are global non-linear stiffness matrices which are linearly and quadratically dependent on the displacement vector, respectively.

The above equation is the nonlinear free vibration equation which can be solved literately as a linear eigenvalue problem assuming that the plate is vibrating in its principal mode in each iteration in which case, this can be expressed as generalized eigenvalue problem as

$$\left[[K]\{q\}-\lambda[M]\right]\{q\}=0 \quad (5)$$

where $\lambda = \omega^2$ with $\varpi$ is the natural frequency of the plate. Eq. (5) is the nonlinear free vibration problem which is random in nature, being dependent on the system properties. Consequently, the natural frequency and mode shapes are random in nature. In deterministic environment, Eq. (5) is evaluated using eigenvalue formulation and solved employing a direct iterative methods, for instance Newton–Raphson methods, incremental methods, etc. However, in random environment, it is not possible to solve the problem using the above mentioned methods without changing the nature of the equation. For this purpose, novel probabilistic procedure developed by the last authors for composite plate Liu et al. [1986]is extended in this work for plate. Based on this, the direct iterative method combined with nonlinear finite element method, i.e., direct iterative-based nonlinear finite element method in conjunction with mean-centred FOPT (DISFEM) with a reasonable accuracy is used to determine the second- order statistics (mean and standard deviation) of nonlinear natural frequency of the plates. 3.1. A DISFEM for nonlinear hygrothermal free vibration problems is termed as solution technique.

### *Solution Technique for Non-linear Hygrothermal Free Vibration Problems*

#### *1) Direct Iterative Technique*

The nonlinear eigenvalue problem as given in Eq. (5) is solved by employing a direct iterative based C0 nonlinear finite element method in conjunction with perturbation technique (DISFEM) assuming that the random changes in eigenvector during iterations does not affect much the nonlinear stiffness matrix Liu et al. [1986].

#### *2) Solutions – First Order Perturbation Technique (FOPT)*

Typical composites are microscopic heterogeneous usually made of fibre and matrix with mechanical and hygrothermal properties varying from one surface to the other. The fabrication and manufacturing of the composites are complicated process due to involvement of large number of uncertainties in their material properties. In the present analysis, the elastic constants such as Young modulus and Poisson's ratio of each constituent material, volume fraction index are treated as independent random variables.

We consider a class of problems where the zero mean random variation is very small when compared to the mean part of random system

properties. In general, without any loss of generality any arbitrary random variable can be represented as the sum of its mean and a zero mean random variable, expressed by superscripts 'd' and 'r', respectively.

A DISFEM approach has been adopted to obtain the second order statistics of dimensionless hygrothermal nonlinear fundamental frequency response to laminated composite plate with randomness in material properties. The material properties are assumed to be basic input random variables. Without any loss of generality, the random variable can be split up as the sum of a mean and a zero random part Liu et al. [1986].

In general, a random variable can be represented as the sum of its mean and zero mean random variable, denoted by superscripts 'd' and 'r', respectively.

$$K = K^d + K^r;\ \lambda_{1i} = \lambda_{1i}{}^d + \lambda_{1i}{}^r, \lambda_{2i} = \lambda_{2i}{}^d + \lambda_{2i}{}^r$$

and $$q_i = q_i{}^d + q_i{}^r \tag{6}$$

where $\lambda_{2i}{}^d = \omega_i{}^{d^2}$, $\lambda_{2i}{}^2 = 2\omega_i{}^d \omega_i{}^r + \omega_i{}^{r^2}$, i= 1,2….,p. The parameter p indicates the size of eigen problem.

Consider a class of problems where the random variation is very small as compared to the mean part of material properties. Further it is quite logical to assume that the coefficient of variation in the derived quantities like $\lambda^r$,$\omega^r$,$q^r$ and $K^r$ are also small as compared to mean values. By substituting and expanding the random parts in Taylor's series keeping the first order terms and neglecting the second and higher order terms, same order of the magnitude term, one can obtain as Kleiber [1992], Yamin [1996].

For hygrothermal nonlinear free vibration analysis

Zeroth order perturbation equation:

$$\left[K^d\right]\left\{q_i{}^d\right\} = \lambda_i{}^d M \left\{q_i{}^d\right\} \tag{7}$$

First order perturbation equation:

$$\left[(K^d - \lambda_i{}^d M)\right]\left\{q_i{}^r\right\} = -(K^r - \lambda_i{}^r M)\left\{q_i{}^d\right\} \text{ or}$$

$$\left[K^d\right]\left\{q_i^r\right\} + \left[K^r\right]\left\{q_i^d\right\} = \lambda_i^r [M]\left\{q_i^d\right\} + \lambda_i^d [M]\left\{q_i^r\right\} \tag{8}$$

Eq. (7) is the deterministic equations related to the mean eigen values and corresponding mean eigenvectors, which can be determined by conventional eigen solution procedures. Eq. (8) the first order perturbation approach is employed in the present study Yamin [1996]. Using this Eq. (8) can be decoupled and the expressions for $\lambda_{1i}{}^r$ and $\lambda_{2i}{}^r$ separate for hygrothermal nonlinear free vibration are obtained. The FEM in conjunction with first order perturbation has been found to be accurate and efficient Yamin [1996]. According to this method, the random variables are expressed by Taylor's series. Keeping the first order terms and neglecting the second and higher-order terms Eq. (6) can be expressed because, the first order is sufficient to yield results with desired accuracy for problems with low variability.

$$\lambda_i^r = \sum_{j=1}^{q} \lambda_{i,j}^d b_j^r;\ \left\{q_i^r\right\} = \sum_{j=1}^{p} q_{i,j}^d b_j^r;\ \left[K^r\right] = \sum_{j=1}^{q} \left[K,_j^d\right] b_i^r; \tag{9}$$

Using the above and decoupled equations, the expressions for $\lambda_{1i,j}^d$ and $\lambda_{2i,j}^d$ are obtained.

Using Eq. (9) the variances of the eigen values can now be expressed as :

$$Var\left(\lambda_{1i}\right) = \sum_{j=1}^{p}\sum_{k=1}^{p} \lambda_{1i,j}^d \lambda_{1i,k}^d Cov\left(b_j{}^r, b_k{}^r\right) \tag{10}$$

$$Var\left(\lambda_{2i}\right) = \sum_{j=1}^{p}\sum_{k=1}^{p} \lambda_{2i,j}^d \lambda_{2i,k}^d Cov\left(b_j{}^r, b_k{}^r\right) \tag{11}$$

where $Cov\left(b_j{}^r, b_k{}^r\right)$ is the cross variance between $b_j{}^r$ and $b_k{}^r$ .The standard deviation (SD) is obtained by the square root of the variance .

## Results and Discussions

The direct iterative based stochastic finite element method (DISFEM) - approach outlined in previous subsection for hygrothermally induced nonlinear free vibration response to nonlinear elastically supported laminated composite plate with random material properties has been presented through numerical examples. The results obtained from the present probabilistic approach have been validated by comparison between the results and those available in literature and independent MCS. Typical results for mean and coefficient of variance {COV(=SD/mean)} of the nonlinear fundamental frequency square referred as fundamental frequency for square plate in the given following text are presented using MATLAB software. A nine nodded Lagrangian isoparametric element with 63degree of freedom per element for the present HSDT model has been used throughout the study. Based on

the convergence study, a (8 × 8) mesh size has been used for numerical computation. Due to the linear nature of variation as mentioned earlier passing through the origin, the results are only presented by COV of the system properties equal to 0.1. Liu [1986] . However, the obtained results revealed that the stochastic approach would be valid up to COV = 0.2 Singh et al. [2001, 2003]. Moreover, the presented results would be sufficient to extrapolate the results for other COV values keeping in mind the limitation of the FOPT.

The basic random variables such as $E_1$, $E_2$, $G_{12}$, $G_{13}$, $G_{23}$, $\upsilon_{12}$, $\alpha_1$, $\alpha_2$, $\beta_2$ , $k_1$, $k_2$ and $k_3$ are sequenced and defined as :

$$b_1 = E_{11},\quad b_2 = E_{22},\quad b_3 = G_{12},\quad b_4 = G_{13},\quad b_5 = G_{23},\quad b_6 = \upsilon_{12},$$
$$b_7 = \alpha_1,\quad b_8 = \alpha_2,\quad b_9 = \beta_2, b_{10} = k_1, b_{11} = k_2, b_{12} = k_3$$

$\beta_1$ is considered to be zero. The following dimensionless foundation stiffness parameters have been used in this study.

$$k_1 = K_1 b^4 / E_{22}^d h^3$$
$$k_2 = K_2 b^2 / E_{22}^d h^3 , k_3 = K_3 b^4 / E_{22}^d h$$

Where, $k_1$, $k_2$ and $k_3$ are the nondimensional Winkler (normal) , Pasternak (shear) and nonlinear Winkler foundation parameters, respectively. The dimensionless linear hygrothermal fundamental frequency $\varpi = (\omega a^2 \sqrt{\rho / E_{22}^d}) / h$ , Nonlinear to linear frequency ratio which is equal to $\varpi_{nl}/\varpi_l$. has been used for analysis. The boundary conditions are for simply supported, clamped and combination of both used for the present analysis

In order to validate the proposed outlined approach the results for mean and standard deviation are compared with those available in literature and an independent Monte Carlo simulation technique.

All edges simply supported (SSSS):

$$v = w = \theta_y = \psi_y = 0, at\ x = 0, a;\quad u = w = \theta_x = \psi_x = 0\ at\ y = 0, b$$

All edges clamped (CCCC):

$$u = v = w = \psi_x = \psi_y = \theta_x = \theta_y = 0, at\ x = 0, a\quad and\ y = 0, b;$$

Two opposite edges clamped and other two simply supported (CSCS):

$$u = v = w = \psi_x = \psi_y = \theta_x = \theta_y = 0, at\ x = 0\quad and\ y = 0;$$
$$v = w = \theta_y = \psi_y = 0, at\ x = a\ \ u = w = \theta_x = \psi_x = 0, at\ y = b;$$

The material properties are used for computation Shen [2001].

and ratio of dimensionless nonlinear to linear fundamental frequency increases when the environmental conditions are changed. On further increase of fiber volume fractions these values further increase, this shows that effects of environmental conditions and fiber volume fractions are significant. It is seen that the present results are in good agreement. ;

*Validation Study: Mean and Standard Deviation*

***1) Validation Study for Mean Fundamental Frequency***

Comparison of dimensionless nonlinear to linear fundamental frequency ($\varpi_{nl}/\varpi_l$.) for perfect $(0^0/90^0)_{2T}$ laminated square plates, plate thickness ratio (a/h=20), aspect ratio ( a/b) =1.0, where dimensionless length of plate (a) =0.1, width of plate (b)=0.1. Initial temperature$T_0$= 25$^0$C, initial moisture $C_0$=0%, T= ($T_0$+ ΔT), C=($C_0$+ ΔC ) where ΔT and ΔC rise in temperature & moisture respectively, simple support S2 under environmental conditions and subjected to biaxial compression is shown in Table. 1.

It is observed that for given fiber volume fraction the dimensionless mean linear fundamental frequency decreases and ratio of dimensionless nonlinear to linear fundamental frequency increases when the environmental conditions are changed. These values further increase with the rise of fiber volume fractions which shows that effects of environmental conditions and fiber volume fractions are significant. It is seen that the present results are in good agreement. It is observed that for given fiber volume fraction the dimensionless mean linear fundamental frequency decreases.

***2) Standard Deviation of Nonlinear Fundamental Frequency***

In this section, the DISFEM results have been compared with that obtained by an independent MCS approach. The normalised standard deviation, i.e. coefficient of variations(COV) for individual random material property $E_{22}$ for cross-ply antisymmetric $(0^0/90^0)_{2T}$ laminated square plates resting on nonlinear (k1=100,k2=00,k3=100) and (k1=100,k2=10,k3=100) elastic foundations, plate thickness

ratio (a/h=20), plate aspect ratio (a/b) =1.0, where length a=0.1, width b=0.1., rise in temperature (ΔT)=100$^0$C, rise in moisture concentration in

percentage ($\Delta C$) =1%, volume fraction (Vf) =0.6, amplitude ratio ( $W_{max}/h$ ) =0.4 with simply supported SSSS( S2) boundary conditions presented in Fig.4.

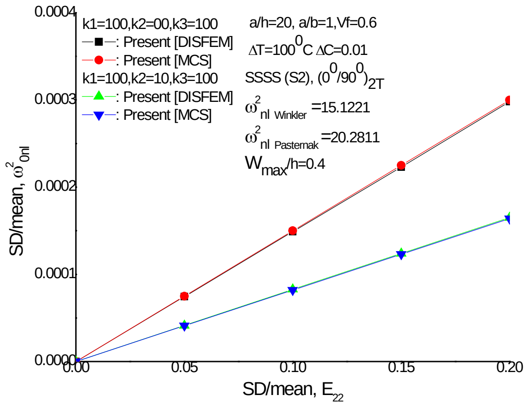


FIG.4. VALIDATION OF PRESENT FOPT RESULTS WITH INDEPENDENT MCS RESULTS OF RANDOM MATERIAL PROPERTIES

It is assumed that the material property $E_{22}$ changes at a time keeping other as "deterministic" with their mean values of material properties. The dashed line is the present [DISFEM] result and the solid line is independent MCS approach. For the MCS approach, the samples are generated using MATLAB to fit the mean and standard deviation. These samples are used in response equation Eq. (6), which is solved repeatedly, adopting conventional eigenvalue procedure, to generate a sample of the hygrothermal post bucking load. The number of samples used for MCS approach is 10,000 based on satisfactory convergence of the results. The normal distribution has been assumed for random number generations in MCS. It can also be observed that the present DISFEM results are in good agreement with MCS results. From the figure it is clear that the DISFEM for present analysis is sufficient to give accurate results for the level of variations considered in the basic random variable. The mean response values obtained by two approaches are almost the same.

*3) Numerical Results: Second Order Statistics of the Nonlinear Fundamental Frequency*

Table 2(a) and 2(b) show the effect of boundary conditions with amplitude ratios ($W_{max}/h$) and input random variables bi{i=1…12, 7-8, 9 and 10..12} on the dimensionless mean ($\varpi_{nl}$) and coefficient of variation ($\omega^2_{nl}$) of the non-linear fundamental frequency of perfect cross ply $(0^0/90^0)_{2T}$ laminated composite square plates resting on non-linear elastic foundations, volume fraction (Vf=0.6), plate thickness ratio ( a/h=20) . It is noticed that clamped supported plates have higher value of dimensionless non-linear mean of the fundamental frequency compared to other supports whereas coefficient of variation of the fundamental frequency is higher for simple support for given conditions. Which shows that clamped supported plates vibrate at higher fundamental frequency compared to other supports. Effects of plate thickness ratios (a/h) with amplitude ratios ($W_{max}/h$) and input random variables bi{i=1…12, 7-8, 9 and 10..12} on the dimensionless mean ($\varpi_{nl}$) and coefficient of variation ($\omega^2_{nl}$) of the non-linear fundamental frequency of perfect cross ply $(0^0/90^0)_{2T}$ laminated composite square plates resting on non-linear elastic foundations, volume fraction (Vf=0.6) is presented in Table 3(a) &(b). It is seen that for given conditions the dimensionless non-linear mean fundamental frequency is significantly affected when the plate is thin compared to moderately thick plates whereas the coefficient of variation of the fundamental frequency is increased for thin plates especially for combined random input variables. It is also noticed that nonlinear foundation stiffness parameters as well are significant compared to without foundations.

Table 4(a) & (b) show the effects of aspect ratio (a/b) with amplitude ratios ($W_{max}/h$) and input random variables bi{i=1…12, 7-8, 9 and 10..12} on the dimensionless mean ($\varpi_{nl}$) and coefficient of variation ($\omega^2_{nl}$) of the nonlinear fundamental frequency of perfect cross ply $(0^0/90^0)_{2T}$ laminated composite square plates resting on non-linear elastic foundations with plate thickness ratio ( a/h=30), volume fraction ($V_f$=0.6). It is seen that for given conditions when the aspect ratios are changed i.e. the plate is rectangular, there is sharp change in dimensionless mean non-linear fundamental frequency for plates resting on nonlinear elastic foundations especially when amplitude ratios are increased. However coefficient of variation of the non-linear fundamental frequency is higher for combined random input variables.

Effects of lay-up with amplitude ratios ($W_{max}/h$) and input random variables bi{i=1…12, 7-8, 9 and 10..12}on the dimensionless mean ($\varpi_{nl}$) and coefficient of variation ($\omega^2_{nl}$) of the non-linear fundamental frequency of perfect laminated composite square plates resting on non-linear elastic foundations with plate thickness ratio ( a/h=20), volume fraction (Vf=0.6) is presented in Table 5(a)&(b).

TABLE.1.

| Lay-up | (Vf) | Environment Conditions | $(\varpi_l)= \omega a^2\sqrt{(\rho_0/E_{22}/h^2)}$ | $(\varpi_{nl}/\varpi_l.)$ | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
| Huang and Zheng [2004] | 0.5 | $\Delta T=0^0C$, $\Delta C=0\%$ | 9.865 | 1.0 | 1.039 | 1.147 | 1.307 | 1.503 |
| | | $\Delta T=200^0C$, $\Delta C=3\%$ | 7.813 | 1.0 | 1.059 | 1.220 | 1.448 | 1.717 |
| Present [HSDT] | 0.5 | $\Delta T=0^0C$, $\Delta C=0\%$ | 9.475 | 1.0 | 1.039 | 1.150 | 1.321 | 1.519 |
| | | $\Delta T=200^0C$, $\Delta C=3\%$ | 7.707 | 1.0 | 1.058 | 1.214 | 1.441 | 1.684 |
| Huang and Zheng [2004] | 0.6 | $\Delta T=0^0C$, $\Delta C=0\%$ | 10.587 | 1.0 | 1.039 | 1.147 | 1.307 | 1.504 |
| | | $\Delta T=200^0C$, $\Delta C=3\%$ | 8.502 | 1.0 | 1.058 | 1.215 | 1.438 | 1.703 |
| Present [HSDT] | 0.6 | $\Delta T=0^0C$, $\Delta C=0\%$ | 10.369 | 1.0 | 1.039 | 1.150 | 1.321 | 1.518 |
| | | $\Delta T=200^0C$, $\Delta C=3\%$ | 8.169 | 1.0 | 1.062 | 1.225 | 1.463 | 1.682 |
| Huang and Zheng [2004] | 0.7 | $\Delta T=0^0C$, $\Delta C=0\%$ | 11.331 | 1.0 | 1.038 | 1.145 | 1.304 | 1.499 |
| | | $\Delta T=200^0C$, $\Delta C=3\%$ | 9.074 | 1.0 | 1.057 | 1.213 | 1.436 | 1.699 |
| Present [HSDT] | 0.7 | $\Delta T=0^0C$, $\Delta C=0\%$ | 11.319 | 1.0 | 1.039 | 1.150 | 1.323 | 1.519 |
| | | $\Delta T=200^0C$, $\Delta C=3\%$ | 8.406 | 1.0 | 1.060 | 1.250 | 1.511 | 1.685 |

TABLE 2 (A)

| BCs | $W_{max}/h$ | $\Delta T=200^0C$, $\Delta C=3\%$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=000,k2=00,k3=000) | | | | | (k1=100,k2=00,k3=100) | | | |
| | | Mean $(\varpi_{nl})$ | COV, $\omega^2_{nl}$ bi | | | | Mean $(\varpi_{nl})$ | COV, $\omega^2_{nl}$ bi | | |
| | | | (i=1.12) | (i=7-8) | (i=9) | (i=10.12) | | (i=1.12) | (i=7-8) | (i=9) | (i=10.12) |
| SSSS | 0.2 | 7.9475 | 0.1580 | 0.0179 | 0.0573 | 0 | 12.1099 | 0.0883 | 0.0077 | 0.0247 | 0.0563 |
| S1 | 0.4 | 9.3952 | 0.1157 | 0.0129 | 0.0412 | 0 | 13.2111 | 0.0754 | 0.0065 | 0.0208 | 0.0475 |
| | 0.6 | 11.0699 | 0.0858 | 0.0094 | 0.0300 | 0 | 14.9088 | 0.0620 | 0.0052 | 0.0165 | 0.0379 |
| | $\varpi_l$ | (7.3899) | | | | | (11.7126) | | | | |
| SSSS | 0.2 | 8.6820 | 0.1173 | 0.0150 | 0.0478 | 0 | 12.5606 | 0.0767 | 0.0071 | 0.0228 | 0.0523 |
| S2 | 0.4 | 10.0142 | 0.0905 | 0.0113 | 0.0361 | 0 | 13.6250 | 0.0662 | 0.0061 | 0.0195 | 0.0446 |
| | 0.6 | 11.9533 | 0.0669 | 0.0081 | 0.0259 | 0 | 15.2657 | 0.0547 | 0.0050 | 0.0159 | 0.0361 |
| | $\varpi_l$ | (8.1692) | | | | | (12.1712) | | | | |
| CCCC | 0.2 | 17.6965 | 0.0759 | 0.0041 | 0.0133 | 0 | 19.9162 | 0.0634 | 0.0033 | 0.0105 | 0.0208 |
| | 0.4 | 18.5755 | 0.0691 | 0.0037 | 0.0118 | 0 | 20.7644 | 0.0585 | 0.0030 | 0.0095 | 0.0192 |
| | 0.6 | 19.9582 | 0.0611 | 0.0032 | 0.0102 | 0 | 22.1013 | 0.0527 | 0.0026 | 0.0083 | 0.0172 |
| | $\varpi_l$ | (17.381) | | | | | (19.6140) | | | | |
| CSCS | 0.2 | 13.5161 | 0.0839 | 0.0069 | 0.0221 | 0 | 16.2816 | 0.0657 | 0.0048 | 0.0152 | 0.0311 |
| | 0.4 | 15.0833 | 0.0684 | 0.0055 | 0.0175 | 0 | 17.6944 | 0.0563 | 0.0040 | 0.0127 | 0.0265 |
| | 0.6 | 17.7494 | 0.0543 | 0.0040 | 0.0127 | 0 | 20.1322 | 0.0471 | 0.0031 | 0.0099 | 0.0208 |
| | $\varpi_l$ | (12.866) | | | | | (15.7120) | | | | |

TABLE 2 (B)

| BCs | $W_{max}/h$ | $\Delta T=200^0C$, $\Delta C=3\%$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=100, k2=10, k3=100) | | | | | (k1=100, k2=10, k3=200) | | | |
| | | Mean $(\varpi_{nl})$ | COV, $\omega^2_{nl}$ bi | | | | Mean $(\varpi_{nl})$ | COV, $\omega^2_{nl}$ bi | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| | 0.2 | 17.6232 | 0.0672 | 0.0036 | 0.0116 | 0.0590 | 17.6495 | 0.0670 | 0.0036 | 0.0116 | 0.0589 |
| SSSS | 0.4 | 18.4428 | 0.0618 | 0.0033 | 0.0107 | 0.0541 | 18.5440 | 0.0613 | 0.0033 | 0.0105 | 0.0536 |
| S1 | 0.6 | 19.8123 | 0.0547 | 0.0029 | 0.0094 | 0.0476 | 20.0337 | 0.0540 | 0.0029 | 0.0091 | 0.0471 |
| | $\varpi_l$ | (17.3491) | | | | | (17.3491) | | | | |
| | 0.2 | 17.6359 | 0.0654 | 0.0036 | 0.0289 | 0.0589 | 17.6623 | 0.0652 | 0.0036 | 0.0115 | 0.0587 |
| SSSS | 0.4 | 18.4575 | 0.0602 | 0.0033 | 0.0106 | 0.0540 | 18.5586 | 0.0596 | 0.0033 | 0.0105 | 0.0535 |
| S2 | 0.6 | 19.8321 | 0.0532 | 0.0029 | 0.0093 | 0.0475 | 20.0362 | 0.0526 | 0.0029 | 0.0092 | 0.0470 |
| | $\varpi_l$ | (17.3540) | | | | | (17.3540) | | | | |
| | 0.2 | 24.0483 | 0.0528 | 0.0021 | 0.0069 | 0.0340 | 24.0668 | 0.0527 | 0.0021 | 0.0340 | 0.0340 |
| CCCC | 0.4 | 24.7317 | 0.0500 | 0.0020 | 0.0064 | 0.0320 | 24.8032 | 0.0498 | 0.0020 | 0.0064 | 0.0318 |
| | 0.6 | 25.9103 | 0.0462 | 0.0018 | 0.0058 | 0.0291 | 26.0626 | 0.0459 | 0.0018 | 0.0058 | 0.0290 |
| | $\varpi_l$ | (23.8222) | | | | | (23.8222) | | | | |
| | 0.2 | 20.7260 | 0.0571 | 0.0028 | 0.0091 | 0.0452 | 20.7496 | 0.0570 | 0.0028 | 0.0090 | 0.0451 |
| CSCS | 0.4 | 21.8297 | 0.0516 | 0.0025 | 0.0081 | 0.0405 | 21.9181 | 0.0513 | 0.0025 | 0.0080 | 0.0402 |
| | 0.6 | 23.5417 | 0.0453 | 0.0022 | 0.0070 | 0.0349 | 23.7135 | 0.0449 | 0.0021 | 0.0068 | 0.0348 |
| | $\varpi_l$ | (20.3141) | | | | | (20.3141) | | | | |

TABLE 3 (A)

| (a/h) | $W_{max}/h$ | $\Delta T=100^0C$, $\Delta C=1\%$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=000,k2=00,k3=000) | | | | | (k1=100,k2=00,k3=100) | | | | |
| | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| 10 | 0.2 | 9.3567 | 0.0569 | 0.0019 | 0.0063 | 0 | 13.3861 | 0.0578 | 9.51e-04 | 0.0031 | 0.0506 |
| | 0.4 | 10.1665 | 0.0503 | 0.0017 | 0.0054 | 0 | 14.0727 | 0.0529 | 8.74e-04 | 0.0028 | 0.0459 |
| | 0.6 | 11.3121 | 0.0466 | 0.0015 | 0.0047 | 0 | 14.2780 | 0.0993 | 2.04e-06 | 6.62e-06 | 3.75e-05 |
| | $\varpi_l$ | (8.9533) | | | | | (13.0684) | | | | |
| 30 | 0.2 | 9.0940 | 0.1273 | 0.0187 | 0.0603 | 0 | 13.1788 | 0.0802 | 0.0089 | 0.0287 | 0.0526 |
| | 0.4 | 10.5469 | 0.0970 | 0.0139 | 0.0449 | 0 | 14.3385 | 0.0689 | 0.0075 | 0.0243 | 0.0446 |
| | 0.6 | 13.3496 | 0.0508 | 0.0039 | 0.0127 | 0 | 16.4421 | 0.0547 | 0.0058 | 0.0187 | 0.0345 |
| | $\varpi_l$ | (8.6002) | | | | | (12.7989) | | | | |
| 40 | 0.2 | 5.5084 | 0.4427 | 0.0906 | 0.2924 | 0 | 10.9237 | 0.1361 | 0.0230 | 0.0743 | 0.0766 |
| | 0.4 | 7.6814 | 0.2319 | 0.0467 | 0.1509 | 0 | 12.3221 | 0.1085 | 0.0182 | 0.0586 | 0.0604 |
| | 0.6 | 11.0863 | 0.1319 | 0.0228 | 0.0735 | 0 | 14.8942 | 0.0772 | 0.0126 | 0.0407 | 0.0420 |
| | $\varpi_l$ | (4.6524) | | | | | (10.4556) | | | | |
| 50 | 0.2 | 4.4870 | 0.1959 | 0.0341 | 0.1108 | 0 | 6.0644 | 0.5408 | 0.1168 | 0.3772 | 0.2485 |
| | 0.4 | 5.3321 | 0.1092 | 0.0067 | 0.0208 | 0 | 8.2698 | 0.2945 | 0.0632 | 0.2039 | 0.1341 |
| | 0.6 | 6.4594 | 0.4496 | 0.1057 | 0.3412 | 0 | 11.7994 | 0.1510 | 0.0317 | 0.1025 | 0.0671 |
| | $\varpi_l$ | (4.2899) | | | | | (5.1665) | | | | |

TABLE 3 (B)

| (a/h) | $W_{max}/h$ | (TID) $\Delta T=100^0C$, $\Delta C=1\%$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=100, k2=10, k3=100) | | | | | (k1=100, k2=10, k3=200) | | | | |
| | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| 10 | 0.2 | 14.2956 | 0.0998 | 2.08e-08 | 6.68e-08 | 2.83e-06 | 14.2956 | 0.0998 | 2.08e-08 | 6.67e-08 | 2.83e-06 |
| | 0.4 | 14.2951 | 0.0998 | 5.74e-08 | 1.84e-07 | 7.95e-06 | 14.2951 | 0.0998 | 5.72e-08 | 1.83e-07 | 7.93e-06 |
| | 0.6 | 14.2946 | 0.0998 | 7.29e-08 | 2.33e-07 | 1.01e-05 | 14.2946 | 0.0998 | 7.26e-08 | 2.32e-07 | 1.01e-05 |
| | $\varpi_l$ | (14.2958) | | | | | (14.2958) | | | | |
| 30 | 0.2 | 18.7439 | 0.0650 | 0.0044 | 0.0142 | 0.0577 | 18.7714 | 0.0648 | 0.0044 | 0.0141 | 0.0575 |
| | 0.4 | 19.6164 | 0.0598 | 0.0040 | 0.0130 | 0.0528 | 19.7220 | 0.0592 | 0.0040 | 0.0128 | 0.0524 |
| | 0.6 | 21.5866 | 0.0542 | 0.0034 | 0.0111 | 0.0457 | 21.6794 | 0.0526 | 0.0034 | 0.0109 | 0.0452 |
| | $\varpi_l$ | (18.4787) | | | | | (18.4787) | | | | |
| 40 | 0.2 | 16.8189 | 0.0860 | 0.0097 | 0.0313 | 0.0717 | 16.8497 | 0.0857 | 0.0097 | 0.0312 | 0.0715 |
| | 0.4 | 17.8016 | 0.0773 | 0.0087 | 0.0280 | 0.0642 | 17.9183 | 0.0764 | 0.0086 | 0.0277 | 0.0635 |
| | 0.6 | 19.7763 | 0.0639 | 0.0071 | 0.0230 | 0.0526 | 20.0153 | 0.0629 | 0.0069 | 0.0224 | 0.0520 |
| | $\varpi_l$ | (16.5099) | | | | | (16.5099) | | | | |
| 50 | 0.2 | 13.6873 | 0.1436 | 0.0229 | 0.0740 | 0.1084 | 13.7251 | 0.1428 | 0.0228 | 0.0736 | 0.1078 |
| | 0.4 | 14.8670 | 0.1225 | 0.0195 | 0.0629 | 0.0921 | 15.0069 | 0.1204 | 0.0191 | 0.0617 | 0.0906 |
| | 0.6 | 17.2106 | 0.0935 | 0.0147 | 0.0476 | 0.0696 | 17.4947 | 0.0911 | 0.0143 | 0.0461 | 0.0682 |
| | $\varpi_l$ | (13.2960) | | | | | (13.2960) | | | | |

TABLE 4 (A)

| (a/b) | $W_{max}/h$ | (TID) $\Delta T=200^0C$, $\Delta C=3\%$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=000,k2=00,k3=000) | | | | | (k1=100,k2=00,k3=100) | | | | |
| | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| 1.0 | 0.2 | 14.8852 | 0.1631 | 0.0232 | 0.0751 | 0 | 7.5621 | 0.2617 | 0.0445 | 0.1421 | 0.1446 |
| | 0.4 | 16.0397 | 2.3584 | 0.6148 | 0.1964 | 0 | 9.3399 | 0.1744 | 0.0293 | 0.0989 | 0.0951 |
| | 0.6 | 20.6487 | 0.2533 | 0.0455 | 0.1455 | 0 | 12.2225 | 0.1125 | 0.0184 | 0.0589 | 0.0565 |
| | $\varpi_l$ | (18.4662) | | | | | (6.8433) | | | | |
| 1.5 | 0.2 | 15.1823 | 0.1145 | 0.0114 | 0.0352 | 0 | 25.4457 | 0.0764 | 0.0041 | 0.0125 | 0.0646 |
| | 0.4 | 17.5095 | 0.0844 | 0.0086 | 0.0266 | 0 | 27.1794 | 0.0677 | 0.0036 | 0.0111 | 0.0568 |
| | 0.6 | 20.8409 | 0.0730 | 0.0072 | 0.0225 | 0 | 29.4067 | 0.0616 | 0.0038 | 0.0118 | 0.0489 |
| | $\varpi_l$ | (14.3006) | | | | | (24.8296) | | | | |
| 2.0 | 0.2 | 28.6458 | 0.0879 | 0.0038 | 0.0110 | 0 | 42.9142 | 0.0999 | 2.33e-06 | 7.36e-06 | 9.71e-06 |
| | 0.4 | 31.4850 | 0.0758 | 0.0032 | 0.0092 | 0 | 42.9103 | 0.0998 | 2.21e-05 | 6.97e-05 | 7.25e-05 |
| | 0.6 | 33.3998 | 0.0652 | 0.0028 | 0.0086 | 0 | 42.9084 | 0.0999 | 1.33e-06 | 4.13e-06 | 2.70e-05 |
| | $\varpi_l$ | (27.2227) | | | | | (42.9161) | | | | |

TABLE 4 (B)

| (a/b) | $W_{max}/h$ | $\Delta T=200^0C$, $\Delta C=3\%$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=100, k2=10, k3=100) | | | | | (k1=100, k2=10, k3=200) | | | |
| | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ | | | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ | | |
| | | | bi | | | | | bi | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| | 0.2 | 13.9362 | 0.1143 | 0.0131 | 0.0418 | 0.0945 | 13.9697 | 0.1137 | 0.0130 | 0.0416 | 0.0941 |
| 1.0 | 0.4 | 15.0390 | 0.0989 | 0.0113 | 0.0360 | 0.0814 | 15.1637 | 0.0974 | 0.0111 | 0.0354 | 0.0803 |
| | 0.6 | 17.0046 | 0.0792 | 0.0090 | 0.0287 | 0.0646 | 17.2552 | 0.0776 | 0.0087 | 0.0278 | 0.0635 |
| | $\varpi_l$ | (13.5458) | | | | | (13.5458) | | | | |
| | 0.2 | 34.7818 | 0.0642 | 0.0022 | 0.0067 | 0.0604 | 34.8498 | 0.0640 | 0.0022 | 0.0067 | 0.0602 |
| 1.5 | 0.4 | 36.1446 | 0.0598 | 0.0020 | 0.0062 | 0.0561 | 36.4090 | 0.0591 | 0.0020 | 0.0061 | 0.0555 |
| | 0.6 | 38.4913 | 0.0608 | 0.0018 | 0.0056 | 0.0502 | 39.0554 | 0.0530 | 0.0019 | 0.0054 | 0.0497 |
| | $\varpi_l$ | (34.3233) | | | | | (34.3233) | | | | |
| | 0.2 | 42.9158 | 0.1000 | 7.15e-08 | 2.24e-07 | 2.55e-06 | 42.9158 | 0.1000 | 7.13e-08 | 2.23e-07 | 2.55e-06 |
| 2.0 | 0.4 | 42.9154 | 0.1000 | 4.69e-08 | 1.44e-07 | 2.01e-06 | 42.9154 | 0.1000 | 4.68e-08 | 1.44e-07 | 2.00e-06 |
| | 0.6 | 42.9150 | 0.1000 | 5.83e-08 | 1.78e-07 | 2.58e-06 | 42.9150 | 0.1000 | 5.82e-08 | 1.78e-07 | 2.58e-06 |
| | $\varpi_l$ | (42.9161) | | | | | (42.9161) | | | | |

TABLE 5 (A)

| Lay-up | $W_{max}/h$ | (TID) $\Delta T=200^0C$, $\Delta C=3\%$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=000,k2=00,k3=000) | | | | | (k1=100,k2=00,k3=100) | | | |
| | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ | | | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ | | |
| | | | bi | | | | | bi | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| | 0.2 | 9.4417 | 0.1043 | 0.0127 | 0.0405 | 0 | 13.1050 | 0.0724 | 0.0066 | 0.0210 | 0.0481 |
| $(0^0/90^0)_s$ | 0.4 | 10.6049 | 0.0849 | 0.0101 | 0.0322 | 0 | 14.0716 | 0.0638 | 0.0057 | 0.0183 | 0.0419 |
| | 0.6 | 12.2755 | 0.0714 | 0.0077 | 0.0245 | 0 | 15.6440 | 0.0545 | 0.0047 | 0.0151 | 0.0344 |
| | $\varpi_l$ | (9.0067) | | | | | (12.7561) | | | | |
| | 0.2 | 8.6820 | 0.1173 | 0.0150 | 0.0478 | 0 | 12.5606 | 0.0767 | 0.0071 | 0.0228 | 0.0523 |
| $(0^0/90^0)_{2T}$ | 0.4 | 10.0142 | 0.0905 | 0.0113 | 0.0361 | 0 | 13.6250 | 0.0662 | 0.0061 | 0.0195 | 0.0446 |
| | 0.6 | 11.9533 | 0.0669 | 0.0081 | 0.0259 | 0 | 15.2657 | 0.0547 | 0.0050 | 0.0159 | 0.0361 |
| | $\varpi_l$ | (8.1692) | | | | | (12.1712) | | | | |
| | 0.2 | 10.3083 | 0.1161 | 0.0110 | 0.0352 | 0 | 13.7753 | 0.0782 | 0.0062 | 0.0197 | 0.0435 |
| $(\pm45^0)_s$ | 0.4 | 11.2410 | 0.0994 | 0.0092 | 0.0295 | 0 | 14.5828 | 0.0708 | 0.0055 | 0.0175 | 0.0390 |
| | 0.6 | 12.5830 | 0.0824 | 0.0074 | 0.0237 | 0 | 15.7808 | 0.0623 | 0.0047 | 0.0150 | 0.0338 |
| | $\varpi_l$ | (9.9517) | | | | | (13.4761) | | | | |
| | 0.2 | 11.3725 | 0.1132 | 0.0087 | 0.0279 | 0 | 14.5891 | 0.0790 | 0.0053 | 0.0170 | 0.0388 |
| $(\pm45^0)_{2T}$ | 0.4 | 12.1523 | 0.1002 | 0.0077 | 0.0245 | 0 | 15.2962 | 0.0725 | 0.0048 | 0.0155 | 0.0354 |
| | 0.6 | 13.3293 | 0.0851 | 0.0064 | 0.0206 | 0 | 16.3896 | 0.0645 | 0.0043 | 0.0136 | 0.0313 |
| | $\varpi_l$ | (11.0848) | | | | | (14.3336) | | | | |

TABLE 5 (B)

| Lay-up | $W_{max}/h$ | $\Delta T=200^0C$, $\Delta C=3\%$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=100, k2=10, k3=100) | | | | | (k1=100, k2=10, k3=200) | | | |
| | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ | | | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ | | |
| | | | bi | | | | | bi | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| | 0.2 | 18.0330 | 0.0632 | 0.0035 | 0.0111 | 0.0564 | 18.0589 | 0.0631 | 0.0035 | 0.0111 | 0.0562 |
| $(0^0/90^0)_s$ | 0.4 | 18.8057 | 0.0586 | 0.0032 | 0.0102 | 0.0521 | 18.9053 | 0.0581 | 0.0032 | 0.0101 | 0.0517 |
| | 0.6 | 20.0773 | 0.0527 | 0.0028 | 0.0091 | 0.0465 | 20.2965 | 0.0520 | 0.0028 | 0.0090 | 0.0462 |
| | $\varpi_l$ | (17.7711) | | | | | (17.7711) | | | | |
| | 0.2 | 17.6359 | 0.0654 | 0.0036 | 0.0289 | 0.0589 | 17.6623 | 0.0652 | 0.0036 | 0.0115 | 0.0587 |
| $(0^0/90^0)_{2T}$ | 0.4 | 18.4575 | 0.0602 | 0.0033 | 0.0106 | 0.0540 | 18.5586 | 0.0596 | 0.0033 | 0.0105 | 0.0535 |
| | 0.6 | 19.8321 | 0.0532 | 0.0029 | 0.0093 | 0.0475 | 20.0362 | 0.0526 | 0.0029 | 0.0092 | 0.0470 |
| | $\varpi_l$ | (17.3540) | | | | | (17.3540) | | | | |
| | 0.2 | 18.9099 | 0.0627 | 0.0032 | 0.0103 | 0.0520 | 18.9345 | 0.0625 | 0.0032 | 0.0102 | 0.0519 |
| $(\pm45^0)_s$ | 0.4 | 19.5117 | 0.0592 | 0.0030 | 0.0096 | 0.0489 | 19.6072 | 0.0587 | 0.0030 | 0.0095 | 0.0485 |
| | 0.6 | 20.4771 | 0.0545 | 0.0028 | 0.0088 | 0.0447 | 20.6812 | 0.0539 | 0.0027 | 0.0086 | 0.0443 |
| | $\varpi_l$ | (18.6988) | | | | | (18.6988) | | | | |
| | 0.2 | 19.4092 | 0.0623 | 0.0030 | 0.0096 | 0.0487 | 19.4331 | 0.0622 | 0.0030 | 0.0096 | 0.0486 |
| $(\pm45^0)_{2T}$ | 0.4 | 19.9700 | 0.0592 | 0.0028 | 0.0091 | 0.0461 | 20.0630 | 0.0587 | 0.0028 | 0.0090 | 0.0458 |
| | 0.6 | 20.8857 | 0.0548 | 0.0026 | 0.0083 | 0.0425 | 21.0854 | 0.0541 | 0.0026 | 0.0082 | 0.0422 |
| | $\varpi_l$ | (19.2148) | | | | | (19.2148) | | | | |

TABLE 6(A)

| (Vf) | $W_{max}/h$ | ΔT=200⁰C, ΔC=3% | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=000,k2=00,k3=000) | | | | | (k1=100,k2=00,k3=100) | | | | |
| | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | |
| | | | (i=1...12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1...12) | (i=7-8) | (i=9) | (i=10..12) |
| | 0.2 | 20.4591 | 0.1412 | 0.0245 | 0.0614 | 0 | 7.6688 | 0.2079 | 0.0433 | 0.1083 | 0.1132 |
| 0.50 | 0.4 | 4.0957 | 0.6244 | 0.1524 | 0.3817 | 0 | 9.1620 | 0.1480 | 0.0305 | 0.0763 | 0.0796 |
| | 0.6 | 7.8946 | 0.1897 | 0.0439 | 0.1100 | 0 | 11.5371 | 0.0975 | 0.0196 | 0.0491 | 0.0510 |
| | $\varpi_l$ | (18.5868) | | | | | (7.1003) | | | | |
| | 0.2 | 20.6708 | 0.1518 | 0.0238 | 0.0675 | 0 | 7.6656 | 0.2296 | 0.0431 | 0.1217 | 0.1255 |
| 0.55 | 0.4 | 3.4654 | 0.9605 | 0.2118 | 0.5989 | 0 | 9.2899 | 0.1588 | 0.0295 | 0.0834 | 0.0858 |
| | 0.6 | 7.9996 | 0.2002 | 0.0428 | 0.1209 | 0 | 11.8654 | 0.1025 | 0.0186 | 0.0526 | 0.0535 |
| | $\varpi_l$ | (18.6723) | | | | | (7.0304) | | | | |
| | 0.2 | 20.8852 | 0.1631 | 0.0232 | 0.0751 | 0 | 7.5621 | 0.2617 | 0.0445 | 0.1421 | 0.1446 |
| 0.60 | 0.4 | 2.0397 | 2.3584 | 0.6148 | 0.1964 | 0 | 9.3399 | 0.1744 | 0.0293 | 0.0989 | 0.0951 |
| | 0.6 | 7.6487 | 0.2533 | 0.0455 | 0.1455 | 0 | 12.2225 | 0.1125 | 0.0184 | 0.0589 | 0.0565 |
| | $\varpi_l$ | (18.4662) | | | | | (6.8433) | | | | |
| | 0.2 | 20.1739 | 0.1933 | 0.0248 | 0.0927 | 0 | 7.3251 | 0.3130 | 0.0481 | 0.1744 | 0.1753 |
| 0.65 | 0.4 | 25.1405 | 0.1272 | 0.0154 | 0.0573 | 0 | 9.2942 | 0.1976 | 0.0301 | 0.1091 | 0.1093 |
| | 0.6 | 6.9549 | 0.3084 | 0.0547 | 0.1985 | 0 | 12.4020 | 0.1280 | 0.0189 | 0.0690 | 0.0623 |
| | $\varpi_l$ | (17.8546) | | | | | (6.4939) | | | | |
| | 0.2 | 19.1924 | 0.2361 | 0.0275 | 0.1202 | 0 | 6.9018 | 0.3996 | 0.0554 | 0.2306 | 0.2289 |
| 0.70 | 0.4 | 25.6547 | 0.1380 | 0.0152 | 0.0704 | 0 | 9.1233 | 0.2324 | 0.0320 | 0.1331 | 0.1315 |
| | 0.6 | 6.1250 | 0.5317 | 0.0814 | 0.3420 | 0 | 12.3794 | 0.1529 | 0.0204 | 0.0859 | 0.0725 |
| | $\varpi_l$ | (16.6251) | | | | | (5.9037) | | | | |

TABLE 6 (B)

| ($V_f$) | $W_{max}/h$ | ΔT=200⁰C, ΔC=3% | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | (k1=100, k2=10, k3=100) | | | | | (k1=100, k2=10, k3=200) | | | | |
| | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | | Mean ($\varpi_{nl}$) | COV, $\omega^2_{nl}$ bi | | | |
| | | | (i=1..12) | (i=7-8) | (i=9) | (i=10.12) | | (i=1..12) | (i=7-8) | (i=9) | (i=10..12) |
| | 0.2 | 13.0927 | 0.1049 | 0.0148 | 0.0371 | 0.0862 | 13.1214 | 0.1045 | 0.0148 | 0.0370 | 0.0858 |
| 0.50 | 0.4 | 14.0715 | 0.0915 | 0.0129 | 0.0323 | 0.0749 | 14.1788 | 0.0903 | 0.0127 | 0.0318 | 0.0739 |
| | 0.6 | 15.8314 | 0.0740 | 0.0103 | 0.0259 | 0.0599 | 16.0551 | 0.0726 | 0.0100 | 0.0252 | 0.0590 |
| | $\varpi_l$ | (12.7568) | | | | | (12.7568) | | | | |
| | 0.2 | 13.5048 | 0.1091 | 0.0139 | 0.0392 | 0.0898 | 13.5356 | 0.1086 | 0.0138 | 0.0390 | 0.0894 |
| 0.55 | 0.4 | 14.5457 | 0.0947 | 0.0120 | 0.0339 | 0.0776 | 14.6603 | 0.0934 | 0.0118 | 0.0334 | 0.0766 |
| | 0.6 | 16.4030 | 0.0762 | 0.0096 | 0.0271 | 0.0619 | 16.6427 | 0.0747 | 0.0093 | 0.0263 | 0.0608 |
| | $\varpi_l$ | (13.1422) | | | | | (13.1422) | | | | |
| | 0.2 | 13.9362 | 0.1143 | 0.0131 | 0.0418 | 0.0945 | 13.9697 | 0.1137 | 0.0130 | 0.0416 | 0.0941 |
| 0.60 | 0.4 | 15.0390 | 0.0989 | 0.0113 | 0.0360 | 0.0814 | 15.1637 | 0.0974 | 0.0111 | 0.0354 | 0.0803 |
| | 0.6 | 17.0046 | 0.0792 | 0.0090 | 0.0287 | 0.0646 | 17.2552 | 0.0776 | 0.0087 | 0.0278 | 0.0635 |
| | $\varpi_l$ | (13.5458) | | | | | (13.5458) | | | | |
| | 0.2 | 14.4031 | 0.1209 | 0.0124 | 0.0451 | 0.1007 | 14.4400 | 0.1203 | 0.0124 | 0.0449 | 0.1002 |
| 0.65 | 0.4 | 15.5688 | 0.1043 | 0.0107 | 0.0387 | 0.0864 | 15.7058 | 0.1026 | 0.0105 | 0.0381 | 0.0851 |
| | 0.6 | 17.6378 | 0.0832 | 0.0085 | 0.0307 | 0.0683 | 17.9127 | 0.0813 | 0.0082 | 0.0298 | 0.0670 |
| | $\varpi_l$ | (13.9831) | | | | | (13.9831) | | | | |
| | 0.2 | 14.9347 | 0.1291 | 0.0118 | 0.0492 | 0.1085 | 14.9760 | 0.1284 | 0.0118 | 0.0490 | 0.1080 |
| 0.70 | 0.4 | 16.1645 | 0.1110 | 0.0101 | 0.0422 | 0.0929 | 16.3216 | 0.1091 | 0.0099 | 0.0414 | 0.0914 |
| | 0.6 | 18.3308 | 0.0883 | 0.0080 | 0.0334 | 0.0733 | 18.6373 | 0.0861 | 0.0078 | 0.0323 | 0.0718 |
| | $\varpi_l$ | (14.4825) | | | | | (14.4825) | | | | |

TABLE 7 (A)

| Environment conditions. | $W_{max}/h$ | Mean $(\varpi_{nl})$ | (k1=000,k2=00,k3=000) | | | | Mean $(\varpi_{nl})$ | (k1=100,k2=00,k3=100) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | COV, $\omega^2_{nl}$ | | | | | COV, $\omega^2_{nl}$ | | | |
| | | | bi | | | | | bi | | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| | 0.2 | 10.7832 | 0.1172 | 0.0013 | 0.0041 | 0 | 14.7316 | 0.0779 | 6.83e-04 | 0.0022 | 0.0459 |
| $\Delta T=0^0C$, $\Delta C=0\%$ | 0.4 | 11.9360 | 0.0978 | 0.0010 | 0.0034 | 0 | 15.7033 | 0.0696 | 6.04e-04 | 0.0020 | 0.0406 |
| | 0.6 | 13.7069 | 0.0778 | 8.07e-04 | 0.0026 | 0 | 17.2449 | 0.0599 | 5.10e-04 | 0.0017 | 0.0342 |
| | $\varpi_l$ | (10.3699) | | | | | (14.3929) | | | | |
| | 0.2 | 10.3865 | 0.0773 | 0.0063 | 0.0205 | 0 | 14.1424 | 0.0618 | 0.0034 | 0.0111 | 0.0456 |
| $\Delta T=100^0C$ | 0.4 | 11.5541 | 0.0643 | 0.0052 | 0.0167 | 0 | 15.1234 | 0.0548 | 0.0030 | 0.0097 | 0.0400 |
| $\Delta C=1\%$ | 0.6 | 13.3496 | 0.0508 | 0.0039 | 0.0127 | 0 | 16.6843 | 0.0466 | 0.0025 | 0.0081 | 0.0334 |
| | $\varpi_l$ | (9.9703) | | | | | (13.8024) | | | | |
| | 0.2 | 9.5488 | 0.0969 | 0.0124 | 0.0395 | 0 | 13.1914 | 0.0695 | 0.0065 | 0.0207 | 0.0474 |
| $\Delta T=200^0C$ | 0.4 | 10.7869 | 0.0780 | 0.0097 | 0.0311 | 0 | 14.2154 | 0.0608 | 0.0056 | 0.0179 | 0.0410 |
| $\Delta C=2\%$ | 0.6 | 12.6350 | 0.0643 | 0.0071 | 0.0199 | 0 | 16.0054 | 0.0523 | 0.0046 | 0.0147 | 0.0329 |
| | $\varpi_l$ | (9.0914) | | | | | (12.8266) | | | | |
| | 0.2 | 8.7616 | 0.1222 | 0.0193 | 0.0608 | 0 | 12.2737 | 0.0793 | 0.0099 | 0.0310 | 0.0491 |
| $\Delta T=300^0C$ | 0.4 | 10.0770 | 0.0946 | 0.0147 | 0.0462 | 0 | 13.3446 | 0.0682 | 0.0084 | 0.0263 | 0.0417 |
| $\Delta C=3\%$ | 0.6 | 12.0108 | 0.0699 | 0.0106 | 0.0332 | 0 | 14.9963 | 0.0560 | 0.0068 | 0.0213 | 0.0335 |
| | $\varpi_l$ | (8.2596) | | | | | (11.8832) | | | | |
| | 0.2 | 7.8747 | 0.1582 | 0.0285 | 0.0869 | 0 | 11.2666 | 0.0929 | 0.0139 | 0.0424 | 0.0515 |
| $\Delta T=400^0C$ | 0.4 | 9.2960 | 0.1161 | 0.0205 | 0.0627 | 0 | 12.3995 | 0.0780 | 0.0115 | 0.0352 | 0.0426 |
| $\Delta C=4\%$ | 0.6 | 11.3154 | 0.0822 | 0.0141 | 0.0432 | 0 | 14.1231 | 0.0625 | 0.0091 | 0.0278 | 0.0334 |
| | $\varpi_l$ | (7.3093) | | | | | (10.8418) | | | | |

TABLE 7 (B)

| Environment conditions. | $W_{max}/h$ | Mean $(\varpi_{nl})$ | ($k_1$=100, $k_2$=10, $k_3$=100) | | | | Mean $(\varpi_{nl})$ | ($k_1$=100, $k_2$=10, $k_3$=200) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | COV, $\omega^2_{nl}$ | | | | | COV, $\omega^2_{nl}$ | | | |
| | | | bi | | | | | bi | | | |
| | | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) | | (i=1…12) | (i=7-8) | (i=9) | (i=10..12) |
| | 0.2 | 20.3455 | 0.0627 | 3.58e-04 | 0.0012 | 0.0533 | 20.3730 | 0.0625 | 3.57e-04 | 0.0012 | 0.0532 |
| $\Delta T=0^0C$, $\Delta C=0\%$ | 0.4 | 21.1029 | 0.0587 | 3.34e-04 | 0.0011 | 0.0497 | 21.2103 | 0.0583 | 3.30e-04 | 0.0011 | 0.0493 |
| | 0.6 | 22.4062 | 0.0532 | 3.00e-04 | 9.74e-04 | 0.0447 | 22.6316 | 0.0526 | 2.94e-04 | 9.55e-04 | 0.0443 |
| | $\varpi_l$ | (20.0972) | | | | | ( 20.0972) | | | | |
| | 0.2 | 19.5143 | 0.0575 | 0.0018 | 0.0058 | 0.0531 | 19.5406 | 0.0573 | 0.0018 | 0.0058 | 0.0530 |
| $\Delta T=100^0C$ | 0.4 | 20.2801 | 0.0536 | 0.0017 | 0.0054 | 0.0494 | 20.3816 | 0.0532 | 0.0017 | 0.0053 | 0.0490 |
| $\Delta C=1\%$ | 0.6 | 21.5927 | 0.0482 | 0.0015 | 0.0048 | 0.0442 | 21.8063 | 0.0477 | 0.0015 | 0.0047 | 0.0502 |
| | $\varpi_l$ | (19.2643) | | | | | (19.2643) | | | | |
| | 0.2 | 18.2044 | 0.0614 | 0.0034 | 0.0109 | 0.0553 | 18.2300 | 0.0612 | 0.0034 | 0.0108 | 0.0552 |
| $\Delta T=200^0C$ | 0.4 | 19.0043 | 0.0568 | 0.0031 | 0.0100 | 0.0509 | 19.1025 | 0.0563 | 0.0031 | 0.0099 | 0.0505 |
| $\Delta C=2\%$ | 0.6 | 20.4032 | 0.0507 | 0.0028 | 0.0088 | 0.0451 | 20.6041 | 0.0502 | 0.0027 | 0.0087 | 0.0448 |
| | $\varpi_l$ | (17.9358) | | | | | (17.9358) | | | | |
| | 0.2 | 16.9859 | 0.0655 | 0.0051 | 0.0162 | 0.0569 | 17.0104 | 0.0654 | 0.0051 | 0.0161 | 0.0568 |
| $\Delta T=300^0C$ | 0.4 | 17.8201 | 0.0600 | 0.0047 | 0.0147 | 0.0519 | 17.9139 | 0.0595 | 0.0046 | 0.0146 | 0.0515 |
| $\Delta C=3\%$ | 0.6 | 19.2130 | 0.0528 | 0.0041 | 0.0129 | 0.0453 | 19.4004 | 0.0522 | 0.0040 | 0.0126 | 0.0450 |
| | $\varpi_l$ | (16.6995) | | | | | (16.6995) | | | | |
| | 0.2 | 15.6784 | 0.0712 | 0.0072 | 0.0219 | 0.0590 | 15.7019 | 0.0710 | 0.0072 | 0.0218 | 0.0588 |
| $\Delta T=400^0C$ | 0.4 | 16.5566 | 0.0645 | 0.0065 | 0.0197 | 0.0531 | 16.6458 | 0.0639 | 0.0064 | 0.0195 | 0.0527 |
| $\Delta C=4\%$ | 0.6 | 17.9968 | 0.0559 | 0.0056 | 0.0170 | 0.0457 | 18.1737 | 0.0552 | 0.0055 | 0.0167 | 0.0453 |
| | $\varpi_l$ | (15.3689) | | | | | (15.3689) | | | | |

It is observed that there is slight increase in dimensionless mean non-linear fundamental frequency for angle ply plates compared to cross ply plates at given conditions. However the coefficient of variation of the non-linear fundamental frequency is also increased for angle ply plates.

Table 6(a) & (b) show the effects of fiber volume fraction ($V_f$) with amplitude ratios ($W_{max}/h$) and input random variables bi{i=1…12, 7-8, 9 and 10..12}on the dimensionless mean ($\varpi_{nl}$) and coefficient of variation ($\omega^2_{nl}$) of the non-linear fundamental frequency of perfect cross ply $(0^0/90^0)_{2T}$ laminated composite square plates resting on non-linear elastic foundations with plate thickness ratio ( a/h=30).

.It is noticed that dimensionless mean non-linear fundamental frequency is significantly affected and

varied when the fiber volume fractions are varied especially for nonlinear elastically supported plates.

However the coefficient of variation of the non-linear fundamental frequency also rises when the fiber volume fractions are increased. Effects of environment conditions with amplitude ratios ($W_{max}$/h) and input random variables bi{i=1…12, 7-8, 9 10..12} on the dimensionless mean ($\varpi_{nl}$) and coefficient of variation ($\omega^2_{nl}$) of the non-linear fundamental frequency of perfect cross ply $(0^0/90^0)_{2T}$ laminated composite square plates resting on non-linear elastic foundations with plate thickness ratio (a/h=20), volume fraction (Vf=0.6), simple Support(S2) boundary conditions is presented in Table 7(a) & (b). It is seen that for given conditions when the environmental conditions are increased the dimensionless mean non-linear fundamental frequency decreases significantly. However the coefficient of variation of the non-linear fundamental frequency increases when there is a rise in temperature and moistures .

## Conclusions

The DISFEM stochastic procedure outlined in the present study has been used to obtain the dimensional mean and COV of the nonlinear fundamental frequency of the nonlinear elastically supported laminated composite plates subjected to uniform change in temperature and moisture with temperature independent material properties. The results have been presented in tabular form for the plate subjected to hygrothermal loadings with the randomness in material properties. The COV of the plate increases as the temperature and moisture increases. This brings out the importance of considering hygrothermal loading as one of the essential parameter from the design point of view especially in aerospace applications where reliability of the components in the presence of hygrothermal loading in the presence of nonlinear elastic foundations is the most important.

The nonlinear natural frequency with the variation in the random variables involved is greatly influenced by side to thickness ratios, plate aspect ratios, and fibre volume fractions, support conditions, nonlinear foundation stiffness parameters and environmental conditions. The COV in nonlinear fundamental frequency is large for combined random input variables compared to individual random input variables for the plates. The effect of volume fraction on the variation in nonlinear natural frequencies dependent strongly on the material compositions and for reliability point of view with minimum fraction should be considered. Moreover, for COV point of view, the clamped support square plates are more desirable as compared to other supports. The effect of randomness of input variables in hygrothermal environment on nonlinear elastically supported laminated composite plates assumes importance for the accountability of uncertainties in the system properties for a reliable and safety design.

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